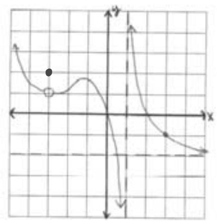
Name______________________________ Calculus

Review

This review sheet should NOT serve as your only review. You should review all notes and tests.

Questions 1 through 7 refer to the graph of $y = f(x)$ shown to the right.



1. $\lim_{x \to 1} f(x) = -\infty$
2. $\lim_{x \to 1} f(x) = \infty$
3. $\lim_{x \to 1} f(x) =$ DNE
4. $\lim_{x \to -3} f(x) = 1$
5. $\lim_{x \to 1} f(x) = -1$
6. $\lim_{x \to -\infty} f(x) = \infty$
7. $\lim_{x \to \infty} f(x) = -2$

For each of the following functions, use the definition of derivative to find $f'(x)$.

Recall: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

8 a) $f(x) = 2x^2 - 8x + 5$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 8(x+h) + 5 - (2x^2 - 8x + 5)}{h} \rightarrow \frac{2(x^2 + 2xh + h^2) - 8x - 8h + 5 - 2x^2 + 8x - 5}{h}$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{8x} - 8h + \cancel{5} - \cancel{2x^2} + \cancel{8x} - \cancel{5}}{h}$$

$$\frac{4xh + 2h^2 - 8h}{h} \rightarrow 2\cancel{h}(2x + h - 4)$$

$$2(2x + h - 4)$$

$$2(2x - 4) \text{ or } 4x - 8$$

b) $f(x) = \sqrt{x+2}$

$$f'(x) = \lim_{h \to 0} \left(\frac{\sqrt{x+h+2} - \sqrt{x+2}}{h}\right)\left(\frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}}\right)$$

$$\frac{x+h+2 - (x+2)}{h(\sqrt{x+h+2} + \sqrt{x+2})} \rightarrow \frac{\cancel{x} + \cancel{h} + \cancel{2} - \cancel{x} - \cancel{2}}{\cancel{h}(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$1\sqrt{x+2} + 1\sqrt{x+2} \rightarrow \frac{1}{2\sqrt{x+2}}$$

Find the derivative of each of the following:

9. $f(x) = 8x + 2\sqrt[3]{x} - \frac{3}{x^3}$

$f(x) = 8x + 2x^{\frac{1}{3}} - 3x^{-3}$

$f'(x) = 8 + \frac{2}{3}x^{-2/3} + 9x^{-4}$

10. $f(x) = \sin(5x^3 + 2x)$

$f'(x) = \cos(5x^3 + 2x)(15x^2 + 2)$

11. $f(x) = \sqrt[3]{(5x^2 + 2x)^2}$

$f(x) = (5x^2 + 2x)^{2/3}$

$f'(x) = \frac{2}{3}(5x^2 + 2x)^{-\frac{1}{3}}(10x + 2)$

12. Find the slope of the line tangent to $y = x(\cos(x))$ when $x = 0$.

$y' = x(-\sin(x)) + \cos(x)(1)$

$= 0(-\sin(0)) + (\cos(0))(1)$

$0 + 1(1) \rightarrow \boxed{1}$

(1,0) 0°

13. Write the equation of the line tangent to $y = 3x^2 - 2x + 1$ when $x = -1$.

$y = 3(-1)^2 - 2(-1) + 1$

$= 3 + 2 + 1 = 6$

$y' = 6x - 2 \rightarrow 6(-1) - 2 \rightarrow -8$

$y - y_1 = m(x - x_1)$

$(-1, 6)$ $\boxed{y - 6 = -8(x+1)}$

14. The following table records the values of $f, f', g,$ *and* g' at $x = 1$, $x = 2$, and $x = 3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

If $n(x) = \boxed{\frac{f(x)}{g(x)}}$, $h(x) = f(g(x))$, find the value of each of the following: a) $n'(2)$ b) $h'(1)$

a) $n'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} \rightarrow \frac{3(4) - (5)(4)}{3^2} \rightarrow \frac{12 - 20}{9} \rightarrow \boxed{\frac{-8}{9}}$

b) $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$= f'g(1) \cdot g'(1)$

$= f'(2) \cdot 3$

$= 4 \cdot 3 = \boxed{12}$

15. If $f(x) = \sqrt[3]{(x^2 - 2x - 1)^2}$, then $f'(0) = ?$

$f(x) = (x^2 - 2x - 1)^{2/3}$

$f'(x) = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{1}{3}} \cdot (2x - 2)$

$f'(0) = \frac{2}{3}(0^2 - 2(0) - 1)^{-\frac{1}{3}} \cdot (2(0) - 2)$

$= \frac{2}{3}(-1)^{-\frac{1}{3}} \cdot (-2)$

$= \frac{-2}{3} \cdot -2 \rightarrow \frac{4}{3}$

$\frac{1}{(-1)^{\frac{1}{3}}} \rightarrow -1$

16. Is $h(x)$ continuous for all real numbers? If so show why.

$$h(x) = \begin{cases} x+3, & x \le -2 \\ -x^2, & x > -2 \end{cases}$$

$h(-2) = 1$

$\lim_{x \to -2} \neq h(-2)$ $\lim_{x \to -2^-} = 1$ $\lim_{x \to -2^+} = -4$

DNE $\neq 1$ $\therefore$ $h(x)$ is not continuous DNE

17. Evaluate $\lim_{x \to 2} \frac{\cancel{x^2+3x-10}}{\cancel{x-2}}$. $(x+5)\cancel{(x-2)}$

$2+5 = \boxed{7}$

18. Evaluate $\lim_{x \to \infty} \frac{2x^3-3}{3x^3+25}$. $= \boxed{\frac{2}{3}}$

19. Find the derivative of the following:

a) $f(x) = \overset{u}{e^{2x}} \overset{v}{\sin(3x)}$

$$f'(x) = e^{2x}((\cos(3x)(3)) + \sin(3x)e^{2x} \cdot 2$$
$$= 3e^{2x}\cos(3x) + 2e^{2x}\sin(3x)$$

b) $y = \frac{\ln(2x)}{\sqrt{x^2+5x}}$

$(x^2+5x)^{\frac{1}{2}}$

$$y' = \frac{\sqrt{x^2+5x}\left(\frac{1}{2x}\right) \cdot 2 - \ln(2x)\frac{1}{2}(x^2+5x)^{-\frac{1}{2}}(2x)}{x^2+5x}$$

$$y' = \frac{\frac{1}{x}\sqrt{x^2+5x} - x\ln(2x)(x^2+5x)^{-\frac{1}{2}}}{x^2+5x}$$